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ON THE THEORY OF SELF-PROTECTED SURFACES

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This paper is concerned with protection of the boundary surface from incident seismic waves incoming to the boundary from below. The main purpose is to study the distribution of the amplitude of oscillations over the boundary with variance of its shape. It is clear that the total incoming kinetic energy along the boundary is the same as the one in the incident wave. The problem of how to suppress the amplitude on certain boundary intervals at the expense of its amplification elsewhere is posed.

1. INTRODUCTION

Many acoustical problems are connected with the need to provide the desired structure of the wave fields in some domains. In noise control, the main efforts aim at optimal reduction of the sound field intensity. Various techniques are applied for this purpose based, from the theoretical point of view, upon the ideas related to Helmholtz resonators (see reference [1]). In architectural acoustics there is a need to minimize the amplitude of the sound wave reflected from the walls of the concert hall. This purpose can be achieved by optimization of the hall shape in total, or by using a special cover for the walls.

Another research direction is related to the following question: is there any efficient way to affect the structure of the reflected acoustic wave by the use of an optimal local structure of the boundary surface itself? In this connection some new methods in the design of the scattered wave should be mentioned as a considerable advance in acoustics of concert halls. Thus, Schroeder [2] proposed a special stepped form of the cover which provides the absence of the specular reflection for the case of normal incidence. Recently, this approach has been adapted for oblique incidence by Feldman [3].

The aim of the present work is to investigate a contiguous problem: can the pressure on the boundary which is subjected to an incident wave be reduced by

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any special choice of the shape of the boundary itself? In other words, can the boundary protect itself from the scattered wave? Such a formulation is urgent for many fields. This is of special interest for seismology, where a long-wave structure of the seismic fields generated by earthquakes involves additional obstacles when arranging a protection method.

It is clear, from the physical point of view, that the incoming energy cannot be dissipated across a boundary surface. Thus, the total kinetic energy over the boundary keeps the same value with arbitrary variation of the shape of the boundary surface. Hence, artificial amplification of the amplitude (and the local wave energy as well) along some boundary regions may suppress these along some others. Below it is shown how to place under these ideas a quantitative mathematical foundation. The results obtained are not very optimistic but worthy of detailed consideration. It is proved, for a number of specific geometries, that the amplitude of the free-surface vibration tends to a trivial value with decreasing frequency. At the same time, a unique regime is discovered, with the period of corrugations being near the wavelength, that permits considerable suppression of the amplitude over some small boundary domains.

2. PROBLEM FORMULATION AND A SIMPLE SOLUTION

Let the plane wave full upon a curved boundary l as shown in Figure 1

$$\varphi^{(in)}(x, y) = \mathrm{e}^{-ikx}. \tag{1}$$

The boundary of the lower half-plane is assumed to be acoustically hard, so the boundary condition is

$$\frac{\partial \varphi}{\partial n} = 0, \qquad (x, y) \in l, \tag{2}$$

where the total wave field is a sum of incident and diffracted waves

$$\varphi = \varphi^{(in)} + \varphi^{(d)}, \qquad \Delta\varphi + k^2\varphi = 0. \tag{3}$$

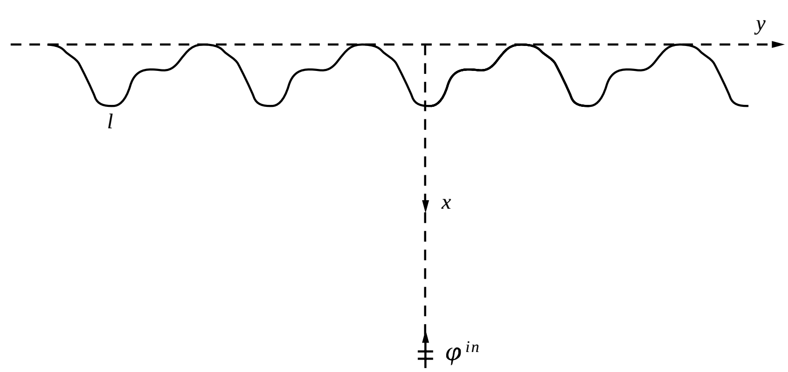

Figure 1. Normal incidence of a plane acoustic wave upon a curved boundary l.

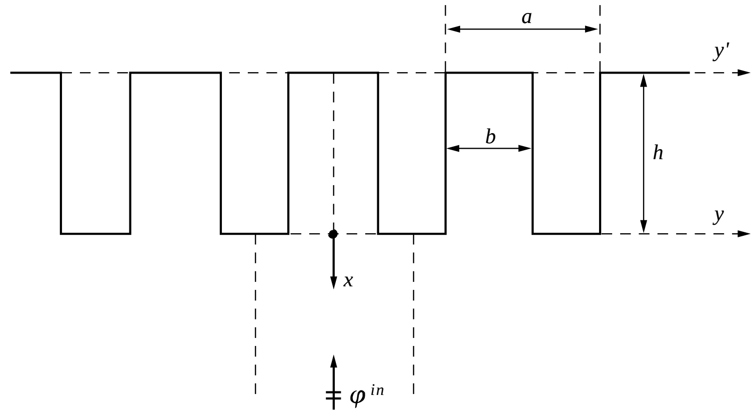

Figure 2. Normal incidence of a plane acoustic wave upon periodic rectangular corrugations.

Here Δ is the Laplace operator, and k is the wave number: $k = \omega/c$. Besides, it is assumed that the wave process is harmonic with respect to time:

$$\tilde{\varphi}(x, y, t) = \varphi(x, y) \exp(-\mathrm{i}\omega t). \tag{4}$$

The physical essence of the function $\varphi(x, y)$ in acoustics is a wave pressure. Another treatment should be applied in seismology where SH shear waves determine $\varphi(x, y)$ as an amplitude of the anti-plane vibrational component $u_z(x, y)$.

The following problem, that is very important in practice, may be posed here: minimize the boundary value of $\varphi(x, y)$, $(x, y) \in l$ selecting an appropriate shape of the free surface $x = x(y)$. From the physical point of view, it is rather unreal to expect any uniform decreasing of the boundary pressure (compared with a case of the plane boundary). Thus, in practice, the problem formulation implies optimal control of the pressure along different parts of the free surface, since its decreasing on the top may appear to be accompanied by an amplification over the "lowland" (or visa versa). Note that the "trivial" value of the amplitude on the plane boundary is well known to be constant and equal to $A_0 \equiv 2$.

A simple solution can be constructed for periodic rectangular corrugations (see Figure 2). Due to the periodicity, the consideration may be restricted by a single layer only. If the period of the profile a is small when compared with the wavelength $\lambda = 2\pi/k$ (so-called "one-mode" case), then in the low-frequency "tube" approximation ($h > a$) the wave field remains plane all over the region and can be represented as follows

$$\varphi_1 = \mathrm{e}^{-\mathrm{i}kx} + R\,\mathrm{e}^{\mathrm{i}kx}, \qquad 0 \leqslant x < \infty, \tag{5}$$

$$\varphi_2 = B \cos kx + D \sin kx, \qquad -h \leqslant x \leqslant 0. \tag{6}$$

The three boundary conditions, to define the three unknown constants R, B, D, are the following ones.

Acoustically hard wall yields

$$\frac{\partial \varphi_2}{\partial x} = 0, \qquad x = -h. \tag{7}$$

The continuity of the pressure inside the layer implies

$$\varphi_1 = \varphi_2, \qquad x = 0. \tag{8}$$

Lastly, mass conservation gives (see reference [1])

$$a\frac{\partial \varphi_1}{\partial x} = b\frac{\partial \varphi_2}{\partial x}, \qquad x = 0. \tag{9}$$

As a result, the following representation for the wave amplitude at the top and the bottom can be derived as

$$A_1 = \varphi_1|_{x=0} = \frac{2a}{a - b\mathrm{i}\tan kh}, \qquad A_2 = \varphi_2|_{x=-h} = \frac{2a}{a\cos kh - b\mathrm{i}\sin kh}. \tag{10, 11}$$

An interesting regime is obtained when $kh \to \pi/2 \sim h \to \lambda/4$ that involves

$$|A_1| \to 0, \qquad |A_2| \to 2\frac{a}{b}. \tag{12}$$

Thus, in this case, protection of the stepped boundary at its lower level can be arranged efficiently.

Therefore, if there is a possibility for a step depth to be of the same order as the wavelength then, accepting $h \sim \lambda/4$, the self-protected boundary can be constructed as a stepped profile. Unfortunately this case is not so attractive for the long-wave practice because its cuttings are too deep.

3. SEMI-ANALYTICAL SOLUTION FOR THE STEPPED PROFILE OF ARBITRARY SIZE

The simplest geometry should be studied in detail, to recognize better if any other corrugations suitable for real practice exist. Rectangular corrugations were thoroughly investigated in the 1950s, for the most part by Russian researchers Deryugin and Myakishev who reduced the problem to an infinite set of linear algebraic equations (for a brief survey, see reference [4]).

If conditions of the one-mode "tube" approximation are broken, then the problem needs a refined treatment, and reduction to an integral equation in the present work is prepared as a more efficient and modern method. Complete representation of the solution is given as

$$\varphi_1(x, y) = \mathrm{e}^{-\mathrm{i}kx} + R\,\mathrm{e}^{\mathrm{i}kx} + \sum_{n=1}^{\infty} B_n\,\mathrm{e}^{-q_n x}\cos\left(2\pi n\frac{y}{a}\right), \qquad x \geqslant 0, \tag{13}$$

$$\varphi_2(x, y) = C_0 \cos k(x+h) + \sum_{n=1}^{\infty} C_n \cosh[r_n(x+h)] \cos\left(2\pi n \frac{y}{b}\right), \qquad -h \leqslant x \leqslant 0, \tag{14}$$

$$q_n = \sqrt{\left(\frac{2\pi n}{a}\right)^2 - k^2}, \qquad r_n = \sqrt{\left(\frac{2\pi n}{b}\right)^2 - k^2}. \tag{15}$$

Formula (14) automatically satisfies the boundary conditions on the acoustically hard boundary: $\partial\varphi_2/\partial x = 0$, $x = -h$, $|y| \leqslant b/2$; $\partial\varphi_2/\partial y = 0$, $y = \pm b/2$, $-h \leqslant x \leqslant 0$.

Introducing the new unknown function $g(y)$ as

$$\left.\frac{\partial\varphi_1}{\partial x}\right|_{x=0} = \begin{cases} (\partial\varphi_2/\partial x)|_{x=0} = g(y), & |y| \leqslant b/2, \\ 0, & b/2 \leqslant |y| \leqslant a/2, \end{cases} \tag{16}$$

one can express the constants R, B_n, C_n in terms of the function $g(y)$. Then the continuity condition (8) leads to the integral equation $g(y)$:

$$\int_{-b/2}^{b/2} K(\eta - y) g(\eta)\, \mathrm{d}\eta = 2, \qquad |y| \leqslant b/2, \tag{17}$$

$$K(y) = -\frac{\cot(kh)}{bk} - \frac{1}{\mathrm{i}ak} + \frac{2}{a}\sum_{n=1}^{\infty} \frac{\cos(2\pi n y/a)}{q_n},$$

$$+\frac{2}{b}\sum_{n=1}^{\infty} \frac{\coth(r_n h)}{r_n} \cos(2\pi n y/b). \tag{18}$$

Obviously, the kernel $K(y)$ possesses a logarithmic singularity at $y \to 0$, due to the following identity [5]

$$\sum_{n=1}^{\infty} \frac{\cos(ny)}{n} = -\ln\left|2 \sin\frac{y}{2}\right|. \tag{19}$$

So any traditional numerical treatment, say the Boundary Elements Technique, may be applied to solve equations (17) and (18). After that, the boundary values of $\varphi(x, y)$ are directly calculated as follows

$$A_1 = \varphi_1|_{x=0} = 2 + \frac{1}{a}\int_{-b/2}^{b/2} g(t)\left\{\frac{1}{k\mathrm{i}} - 2\sum_{n=1}^{\infty} \frac{\cos[2\pi n(t-y)/a]}{q_n}\right\} \mathrm{d}t, \qquad \frac{b}{2} \leqslant |y| \leqslant \frac{a}{2}, \tag{20}$$

$$A_2 = \varphi_2|_{x=-h} = \frac{1}{b}\int_{-b/2}^{b/2} g(t)\left\{2\sum_{n=1}^{\infty}\frac{\cos[2\pi n(t-y)/b]}{r_n \sinh(r_n h)} - \frac{1}{k\sin kh}\right\}\mathrm{d}t, \qquad |y| \leqslant \frac{b}{2}. \tag{21}$$

In the case when $ak \ll 1$, some analytical treatment is possible. For this long-wave regime the kernel $K(y)$ in equation (18) can be simplified as follows

$$K(y) = K_0(y) - \frac{\cot(kh)}{bk} - \frac{1}{\mathrm{i}ak}, \qquad K_0(y) \approx -\frac{1}{\pi}\left[\ln\left|2\sin\frac{\pi y}{a}\right| + \ln\left|2\sin\frac{\pi y}{b}\right|\right]. \tag{22}$$

Let the function $v(x)$ denote a solution of the integral equation

$$\int_{-b/2}^{b/2} K_0(\eta - y)v(\eta)\,\mathrm{d}\eta = 1, \qquad |y| \leqslant b/2, \tag{23}$$

that is evidently independent upon the wave number k. Then the solution of equation (17) is

$$g(y) = v(y)\left\{2 + V\left[\frac{\cot(kh)}{bk} + \frac{1}{\mathrm{i}ak}\right]\right\}, \tag{24}$$

where

$$G = \int_{-b/2}^{b/2} g(y)\,\mathrm{d}y, \qquad V = \int_{-b/2}^{b/2} v(y)\,\mathrm{d}y. \tag{25}$$

Equation (24) involves the following identity regarding the quantities G and V:

$$G = \frac{2V}{1 - \left[\dfrac{\cot(kh)}{bk} + \dfrac{1}{\mathrm{i}ak}\right]V}, \tag{26}$$

with the value of V being independent of k.

Formulas (20) and (21) with $ak \to 0$ give

$$A_1 \sim 2 + \frac{G}{\mathrm{i}ak} = 2 + \frac{2V/\mathrm{i}ak}{1 - \left[\dfrac{\cot(kh)}{bk} + \dfrac{1}{\mathrm{i}ak}\right]V} \to 2, \tag{27}$$

$$A_2 \sim -\frac{G}{bk\sin(kh)} = -\frac{2V/bk\sin(kh)}{1 - \left[\dfrac{\cot(kh)}{bk} + \dfrac{1}{\mathrm{i}ak}\right]V} \to 2. \tag{28}$$

Hence, no protection is possible using the geometry of fixed sizes when $k \to 0$, that is quite natural from the physical point of view. Generally this property is proved in the next section.

At the same time, a direct numerical technique was developed to solve the integral equation (17) for arbitrary combinations of geometrical and physical parameters. Omiting routine details, it is noted that it is based on the Boundary Element Method, that guarantee stable calculations provided integration of the logarithmic singularity of the kernel is performed explicitly. We discovered an exceptional regime $ka \sim 2\pi$ (with $kh \ll 1$) which corresponds to an extreme value of ka in the one-mode process. In a small vicinity of this exclusive combination of the parameters (that is equivalent to $a \sim \lambda$), the vibrational amplitude $\varphi(x, y)$, $(x, y) \in l$ can differ considerably from the trivial value $A_0 = 2$, into various degrees depending on the location of a point on the boundary. Figure 3 demonstrates a geometry where the boundary amplitude over the top is two times more intensive when compared with the case of a plane boundary surface (i.e., $|A_2| \sim 4$). Thus, the amplitude increases by $20 \lg 2 \approx 6$ dB. At the same time, on the lowlands there is a considerable suppression of the amplitudes, and Figure 3 reflects the frequency dependence of the amplitude only at the central point. As follows from the graph, the level of the "gain" becomes considerable only in a very restricted frequency interval where $|\lambda/a - 1| < 0{\cdot}05$. Thus, the results are not so optimistic for practice.

Note that in the typical situation in seismic practice, when the period $T = 1$ s, $c = 1000$ m/s, $\lambda = 1000$ m, the value $h/\lambda = 0{\cdot}02$ corresponds to the step depth $h = 20$ m. It is interesting to note also that increasing the earthquake magnitude by a unit value is equivalent to the boundary amplitude growth by $10 \lg (10^{1{\cdot}5}) = 15$ dB ([6], section 2.2).

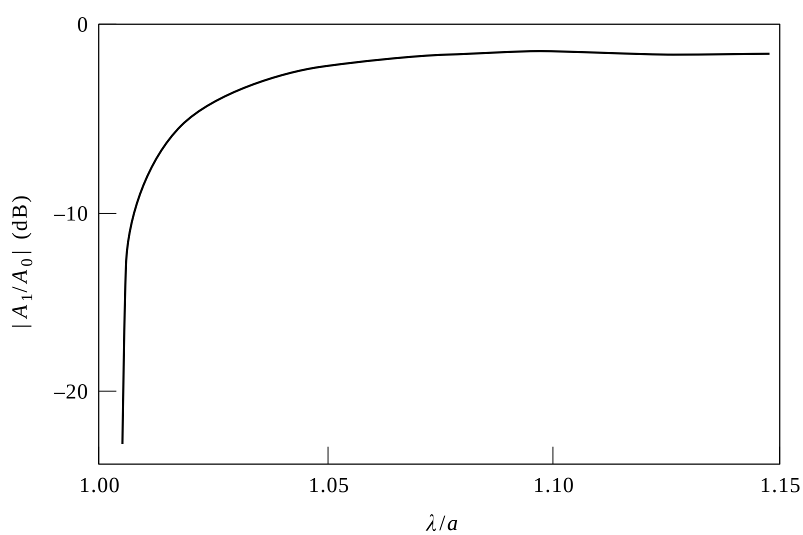

Figure 3. Spectral dependence of the gain $|A_1/A_0|$, $A_0 = 2$ at the point with $x = 0$, $y = b$: $b = 0{\cdot}5a$; $h/a = 0{\cdot}02$.

4. BOUNDARY INTEGRAL EQUATION FOR ARBITRARY CONTOUR l

For arbitrary acoustically hard boundary l given by the following equation (see Figure 1)

$$\left.\frac{\partial\varphi}{\partial n}\right|_l = 0, \qquad l\colon x = x(y), \qquad -\infty < y < \infty, \tag{29}$$

the scattering problem can be reduced, with the use of Kirchhoff formula, to an integral equation over the boundary l [1]

$$\varphi(x, y) - 2\int_l \varphi(\xi, \eta)\frac{\partial\Phi}{\partial n}\,\mathrm{d}l = 2\,\mathrm{e}^{-\mathrm{i}kx}, \qquad (x, y)\in l,$$

$$\Phi = \frac{\mathrm{i}}{4}\,\mathrm{H}_0^{(1)}(kr), \qquad r = \sqrt{(\xi - x)^2 + (\eta - y)^2}, \tag{30}$$

regarding the value of the total wave potential $\varphi(x, y)$ on the boundary contour. It is assumed that the function $x(y)$ is bounded: $|x(y)| \leqslant B$. Function $\mathrm{H}_0^{(1)}$ in equation (30) denotes the Hankel function of the first kind: $\bar{\mathbf{n}}(\xi, \eta)$ is the unit normal-vector at the point $(\xi, \eta)\in l$ directed into the medium.

In the case of the plane boundary, equation (30) directly yields the well known boundary value $\varphi(x, y) \sim 2$. Indeed if $\xi = x \equiv \text{const}$, $\bar{\mathbf{n}} = \{1, 0\}$, then $\partial\Phi/\partial n = (\partial\Phi/\partial r)(\partial r/\partial n) = (\partial\Phi/\partial r)(\partial r/\partial\xi) = (\partial\Phi/\partial r)[(\xi - x)/r] = 0$, so $\varphi(x, y) = 2\exp(-\mathrm{i}kx)$, $(x, y)\in l$ that is twice as intensive when compared with the incident wave.

For arbitrary contour l it is clear, from the physical point of view, that in the low-frequency regime $(k|x(y)| \ll 1)$ the last property remains valid. More exactly, the free term on the left side of equation (30) can considerably deviate from the trivial value $\varphi(x, y) = 2\exp(-\mathrm{i}kx)$ if and only if the integral in (30) becomes significant when the function $\varphi(\xi, \eta) = 2\exp(-\mathrm{i}k\xi)$ is substituted into the integral (30). The result of the substitution can be estimated as follows. Let the point (x, y) belong to the contour l, and let l_1 denote a straight line crossing this point and being parallel to the ground horizon $x = 0$. Then the Green formula involves [1]

$$\left(\int_l - \int_{l_1}\right)\mathrm{e}^{-\mathrm{i}k\xi}\frac{\partial\Phi}{\partial n}\,\mathrm{d}l = \left(\iint_{D_+} - \iint_{D_-}\right)\left[\mathrm{e}^{-\mathrm{i}k\xi}\Delta\Phi + \frac{\partial(\mathrm{e}^{-\mathrm{i}k\xi})}{\partial\xi}\frac{\partial\Phi}{\partial\xi}\right]\mathrm{d}\xi\,\mathrm{d}\eta, \tag{31}$$

where D_+ (D_-) is a union of the set of domains bounded by l_1 from above (below) and by l from below (above). The Green function $\Phi(x, y, \xi, \eta)$ given by equation (30) satisfies the Helmholtz equation (3) and the integral over l_1 in equation (31) has been previously shown to vanish, thus the last formula involves

$$\int_l \mathrm{e}^{-\mathrm{i}k\xi}\frac{\partial\Phi}{\partial n}\,\mathrm{d}l = \frac{k^2}{4}\left(\iint_{D_+} - \iint_{D_-}\right)\mathrm{e}^{-\mathrm{i}k\xi}\left[\mathrm{H}_1^{(1)}(kr)\frac{\xi - x}{r} - \mathrm{i}\mathrm{H}_0^{(1)}(kr)\right]\mathrm{d}\xi\,\mathrm{d}\eta. \tag{32}$$

By applying integration by parts to the first term inside the last brackets, with respect to the ξ-variable, one comes to the following representation

$$\int_l \mathrm{e}^{-\mathrm{i}k\xi} \frac{\partial \Phi}{\partial n}\, \mathrm{d}l = \frac{k}{4} \int_{-\infty}^{\infty} \langle \mathrm{H}_0^{(1)}\{k\sqrt{[\xi(\eta) - x]^2 + (\eta - y)^2}\}$$

$$- \mathrm{e}^{-\mathrm{i}kx}\mathrm{H}_0^{(1)}(k|\eta - y|)\rangle\, \mathrm{d}\eta \sim \mathrm{i}\frac{k^2}{4}\int_{-\infty}^{\infty} [x - \xi(\eta)]\mathrm{H}_0^{(1)}(k|\eta - y|)\, \mathrm{d}\eta, \quad k \to 0, \quad (33)$$

where the functions $\xi = \xi(\eta)$, $|\eta| < \infty$ and $x = x(y)$, $|y| < \infty$ describe the same boundary contour l.

Equation (30) may be rewritten as a functional equation

$$(I - K)\varphi = f, \qquad f = 2\,\mathrm{e}^{-\mathrm{i}kx}, \quad (34)$$

with a small operator K. The Neumann series for its solution

$$\varphi = (I - K)^{-1} f = f + Kf + K^2 f + \cdots \quad (35)$$

involves the first-order approximation as

$$\varphi|_l \approx f + Kf = 2\,\mathrm{e}^{-\mathrm{i}kx} + \mathrm{i}k^2 \int_{-\infty}^{\infty} [x - \xi(\eta)]\mathrm{H}_0^{(1)}(k|\eta - y|)\, \mathrm{d}\eta$$

$$= 2\,\mathrm{e}^{-\mathrm{i}kx} + 2\mathrm{i}kx - \mathrm{i}k^2 \int_{-\infty}^{\infty} \xi(\eta)\mathrm{H}_0^{(1)}(k|\eta - y|)\, \mathrm{d}\eta. \quad (36)$$

The last representation permits estimation of the boundary pressure for small k ($kx \ll 1$) as

$$A = |\varphi(x, y)|_l = 2 + O(k^2), \qquad k \to 0, \quad (x, y) \in l. \quad (37)$$

At the same time, numerical calculations, carried out following equation (36), show that the last formula is a good approximation not only for extremely small kx.

The main general conclusion which may be drawn from the preceding consideration is that in the long-wave regime suppression of the boundary, pressure cannot be provided by any structure of fixed geometry. Thus, self-protection is attainable only when the period of corrugations is coupled with the wave number k. This question is worthy of more detailed discussions.

Obviously, contribution of the integration over any finite part of the infinite interval into the integral in equation (36) is small for small kx. Hence, variation of the boundary shape within any interval of a finite length does not influence the vibrational amplitude $\varphi|_l$ (when $kx \ll 1$). Considerable contribution into the integral (36) may be provided only by distant parts of the boundary. For large η, the function $H_0^{(1)}$ possesses the asymptotics

$$H_0^{(1)}(k|\eta - y|) \sim \sqrt{\frac{2}{\pi k|\eta - y|}}\, e^{i(k|\eta - y| - \pi/4)}. \tag{38}$$

Its real and imaginary parts are sign-alternating functions with the same wavelength $\lambda = 2\pi/k$. Therefore, if the boundary contour $\xi = \xi(\eta)$ is chosen as a periodic function with a period Λ close to λ: $\Lambda \sim \lambda$, then one comes to an integral of some functions of almost constant signs. Such an integral may become arbitrarily large, which confirms the results of the previous section obtained for a particular structure. So any periodic boundary contour l with the period around the wavelength may provide self-protection.

In practice, a finite number N of the boundary ‘‘humps and cavities’’ of various shapes may be sufficient for this purpose. The greater N, the greater the suppression of the surface stress near $\lambda \sim \Lambda$, but with less gain out of a small neighbourhood of the exceptional value: $|\lambda/\Lambda - 1| < \delta \ll 1$. The optimal choice of N is a few tens that provides a good suppression and sufficiently wide range of the frequency parameter where the gain is considerably perceptible. An example of such a type of structure is shown in Figure 4. To widen the spectral interval of the gain, a choice of a finite number of hills and cavities of quasi-constant

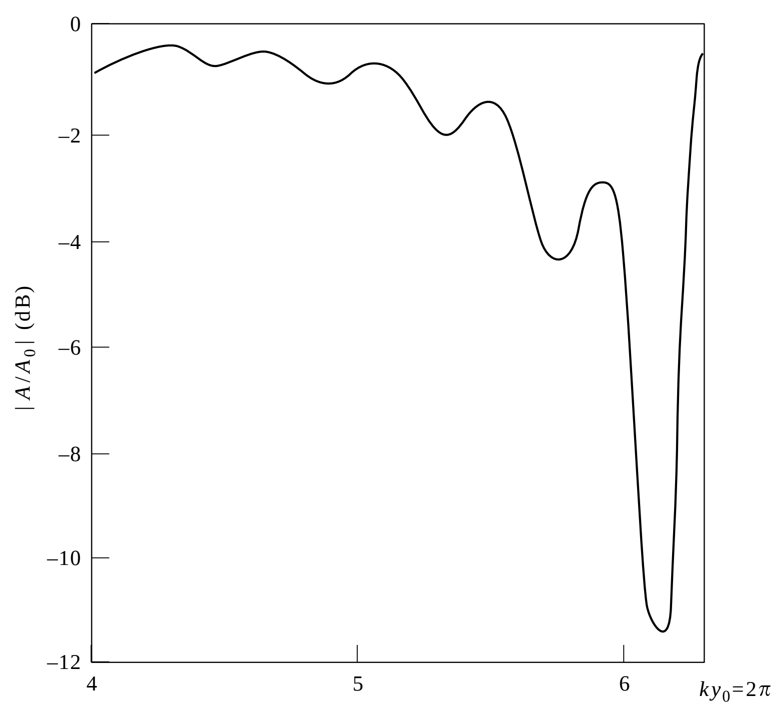

Figure 4. Spectral dependence of the gain for a finite number $N = 30$ of cosine corrugations at the point with $x = 0$, $y = h$: $x = h\cos(\chi y)$; $h/\Lambda = 0{\cdot}03$; $\Lambda = y_0 = 2\pi/\chi$, the period of corrugations.

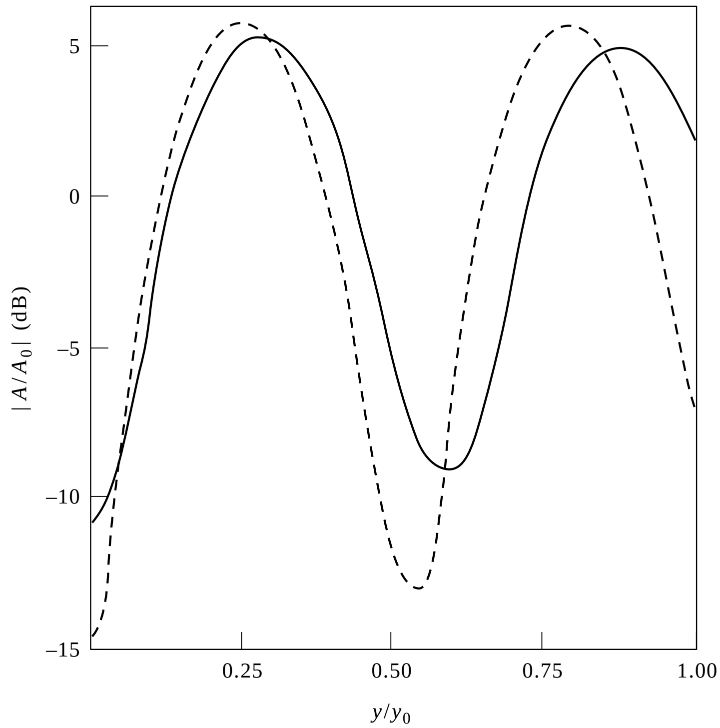

Figure 5. Distribution of the gain along the first period for a finite number of quasi-periodic cosine corrugations:

$$x = \cos\sqrt{(ky)^2 + z^2};\ h/\Lambda = 0{\cdot}03;\ \Lambda = 2\pi/k;\ y_0 = \sqrt{(2\pi + z)^2 - z^2}/k;$$

——, $N = 30$, $z = 5{\cdot}65$; —–, $N = 40$, $z = 7{\cdot}54$.

lengths seems to be appropriate too, as in filtering of surface acoustic waves [7]. A respective example is reflected in Figure 5.

5. BOUNDARY INTEGRAL EQUATION FOR ARBITRARY PERIODIC CONTOUR l

To arrange efficient calculations for any boundary among the predicted class of contours l which can provide the self-protection property, one needs to construct a numerical algorithm to solve equation (30). For arbitrary contours this problem is not so simple, because it takes too large a finite part of l to be accounted for and too many nodes on a grid. However, the results reflected in Figures 4 and 5 have been obtained with the use of a direct numerical treatment on a PC Pentium-133. The aim of the present section is to demonstrate how the main equation (30) can be reduced to a single-period interval when the boundary surface is strictly periodic. In this case, calculations can be performed without any obstacle.

Let l_0 denote a single interval of periodicity with $(x, y) \in l_0$, and a be a period of the corrugations. Then

$$\int_l \varphi \frac{\partial \mathrm{H}_0^{(1)}}{\partial n} \, \mathrm{d}l = -k \int_l \varphi \mathrm{H}_1^{(1)}(kr) \frac{\partial r}{\partial n} \, \mathrm{d}l = -k \int_l \varphi \mathrm{H}_1^{(1)}(kr)(\mathrm{grad}\, r \cdot \bar{\mathbf{n}}) \, \mathrm{d}l$$

$$= -k \sum_{m=-\infty}^{\infty} \int_{l_0} \varphi \mathrm{H}_1^{(1)}(kr_m) \frac{(\bar{\mathbf{r}}_m \cdot \bar{\mathbf{n}})}{r_m} \, \mathrm{d}l, \tag{39}$$

where

$$\bar{\mathbf{r}}_m = \{\xi - x, \eta + am - y\}, \qquad r_m = \sqrt{(\xi - x)^2 + (\eta + am - y)^2}. \tag{40}$$

Hence, one comes to the following integral equation over the single-period interval l_0:

$$\varphi(x, y) + \int_{l_0} \varphi(\xi, \eta) K_1(\xi, \eta, x, y) \, \mathrm{d}l = 2\, \mathrm{e}^{-\mathrm{i}kx}, \qquad (x, y) \in l_0, \tag{41}$$

$$K_1(\xi, \eta, x, y) = \frac{\mathrm{i}k}{2} \sum_{m=-\infty}^{\infty} \mathrm{H}_1^{(1)}(kr_m) \frac{(\bar{\mathbf{r}}_m \cdot \bar{\mathbf{n}})}{r_m}. \tag{42}$$

Convergence of the last series is very slow. However it can be accelerated by extracting the main asymptotic term at $|m| \to \infty$ which yields the series of the type

$$\sum_{m=1}^{\infty} \exp(\pm \mathrm{i}akm)/\sqrt{m}.$$

It can be evaluated for small k by the following method. The Hurwitz formula [8] for generalized zeta-function

$$\zeta(s, a) = \frac{2\Gamma(1-s)}{(2\pi)^{1-s}} \left[\sin \frac{\pi s}{2} \sum_{m=1}^{\infty} \frac{\cos(2\pi m a)}{m^{1-s}} + \cos \frac{\pi s}{2} \sum_{m=1}^{\infty} \frac{\sin(2\pi m a)}{m^{1-s}} \right] \tag{43}$$

is equivalent to the pair of non-oscillating series

$$\sum_{m=1}^{\infty} \frac{\cos(mx)}{m^{\beta}} = x^{\beta-1} \Gamma(1-\beta) \sin \frac{\pi \beta}{2} + \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m)!} \zeta(\beta - 2m) x^{2m}, \tag{44a}$$

$$\sum_{m=1}^{\infty} \frac{\sin(mx)}{m^{\beta}} = x^{\beta-1}\Gamma(1-\beta)\cos\frac{\pi\beta}{2} + \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)!}\zeta(\beta-2m+1)x^{2m-1}, \tag{44b}$$

which involves an evident efficient expression valid for small x and $\beta = 0{\cdot}5$.

6. CONCLUSIONS

From the previous consideration, one can draw the following conclusions.

The low-frequency influence of the incident wave on the boundary of arbitrary fixed geometry is the same as for the case of a plane boundary surface. It means $|\varphi|_l \to 2$ with $k \to 0$.

In order to protect the boundary surface from the incident wave, its shape has to be almost-periodic, with the period being round the wavelength.

The periodic boundaries of various shapes are acceptable for this purpose. The form of different surfaces may involve different gains, but qualitatively, arbitrary almost-periodic function is suitable to provide protection.

For such boundaries, a certain part of its period interval is subjected to more intensive vibrations when compared with the trivial value $A_0 = 2$. However, the gain along remaining intervals is always higher than the loss value.

To reduce the vibrational amplitude at a boundary domain, it is sufficient to take a finite number of "hills". The more periods Λ are taken, the higher is the gain in a small frequency interval $|\Lambda/\lambda - 1| < \delta$, but the more narrow.

Unfortunately considerable suppression of the surface oscillations is possible in very restricted frequency interval only. So the problem needs further investigations, in order to make results more attractive for practice.

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